



Inhomogeneous chiral phases in two-flavor quark matter

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Abstract We present a systematic study of the phase structure of QCD in a generalized Ginzburg-Landau framework. We find, going up in density, a strongly interacting matter might go through the “pion crystal”, an exotic inhomogeneous chiral phase before reaching the full restoration of symmetry.

Key words Quark matter; Chiral symmetry; Solitonic chiral condensate

The possibility of spatially inhomogeneous realizations of chiral symmetry breaking in quark matter has recently been the subject of extensive research [1]. We here report our recent study [2, 3] on the inhomogeneous chiral phases near the (tri-)critical point, (T)CP hereafter. A particular focus is put on the impact of isospin asymmetry since up and down quarks are not equally populated in a realistic situation such as expected in the interior of compact stars. The method we use is the generalized Ginzburg-Landau (gGL) approach. We show that in some region of phase diagram, quark matter may be realized in a form of solitonic charged pion crystal (SPC).

The gGL potential at the minimal order describing the (T)CP at $\mu_I = 0$ is [2]

$$\begin{aligned} \omega(\mathbf{x}) = & -h\sigma + \frac{\alpha_2}{2}\phi^2 + \frac{\alpha_4}{4}(\phi^4 + (\nabla\phi)^2) \\ & + \frac{\alpha_6}{6}\left(\phi^6 + 3[\phi^2(\nabla\phi)^2 - (\phi \cdot \nabla\phi)^2] + 5(\phi \cdot \nabla\phi)^2 + \frac{1}{2}(\Delta\phi)^2\right), \end{aligned} \quad (1)$$

where we utilized the chiral four-vector notation $\phi(\mathbf{x}) = (\sigma(\mathbf{x}), \pi_1(\mathbf{x}), \pi_2(\mathbf{x}), \pi_3(\mathbf{x}))$ with $\sigma(\mathbf{x}) \sim \langle \bar{q}q \rangle$ the scalar condensate and $\pi_a(\mathbf{x}) \sim \langle \bar{q}i\gamma_5\tau_a q \rangle$ ($a = 1, 2, 3$) the pseudoscalar condensates. h is an external field due to a small current quark mass, by which the chiral

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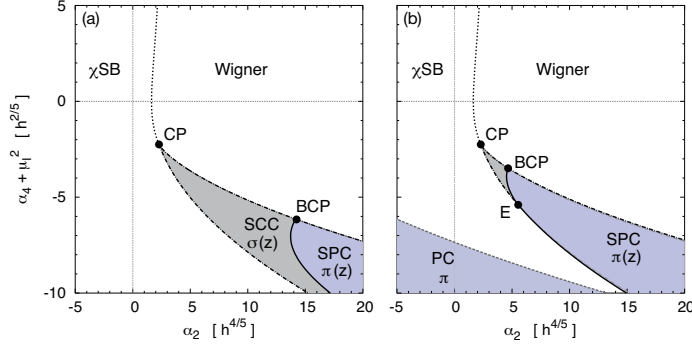


Fig. 1 The gGL phase diagram for nonvanishing isospin densities: (a) for $\mu_I^2 = 0.01$, and (b) for $\mu_I^2 = 0.1$.

O(4) symmetry is broken to its diagonal subgroup, the isospin O(3). The effect of isospin asymmetry can be accommodated via introducing the isospin chemical potential μ_I . It further breaks O(3) to U(1) symmetry, the rotation about the isospin third axis. The terms that should be added are summarized as

$$\delta\omega(\mathbf{x}) = \frac{\beta_2}{2}\pi_\perp^2 + \frac{\beta_4}{4}\pi_\perp^4 + \frac{\beta_{4b}}{4}(\phi^2 - \pi_\perp^2)\pi_\perp^2 + \frac{\beta_{4c}}{4}(\nabla\pi_\perp)^2, \quad (2)$$

where we have defined the charged pion doublet $\pi_\perp = (\pi_1, \pi_2)$. If the system is in the charged pion condensate (PC) with $|\pi_\perp| \neq 0$, that is when U(1) symmetry is spontaneously broken. Within the expansion in μ_I , we find $\beta_2 = -\frac{1}{2}\mu_I^2\alpha_4$, $\beta_4 = -2\mu_I^2\alpha_6$, $\beta_{4b} = -2\mu_I^2\alpha_6$, and $\beta_{4c} = -\frac{4}{3}\mu_I^2\alpha_6$. The gGL potential $\omega(\mathbf{x}) + \delta\omega(\mathbf{x})$ is characterized by five parameters, α_2 , α_4 , α_6 , h , and μ_I^2 . Assuming $\alpha_6 > 0$ for the stability we can replace α_6 with 1 adopting the convention that every quantity with an energy dimension is to be measured in the unit $\alpha_6^{-1/2}$. Then via scaling $\mu_I \rightarrow h^{1/5}\mu_I$, $\phi \rightarrow h^{1/5}\phi$, $\mathbf{x} \rightarrow \mathbf{x}^{-1/5}$, $\alpha_2 \rightarrow \alpha_2 h^{4/5}$, $\alpha_4 \rightarrow \alpha_4 h^{2/5}$, we can get rid of h in ω apart from an overall factor $h^{6/5}$. Accordingly we are left with three parameters α_2 , α_4 and μ_I^2 which are to be measured in units $h^{4/5}$, $h^{2/5}$ and $h^{2/5}$ respectively.

Now we can evaluate the phase structure in the (α_2, α_4) -plane. The result is displayed in Fig. 1; (a) is for $\mu_I^2 = 0.01$, and (b) is for $\mu_I^2 = 0.1$. The former roughly corresponds to $\mu_I \sim 50$ MeV, and the latter to $\mu_I \sim 150$ MeV [3]. The vertical axis is shifted by μ_I^2 so as to make the trivial shift of location of CP invisible. The shift corresponds to higher μ and lower T [3]. The chiral symmetry is broken in the χ SB phase while it is nearly restored in the “Wigner” phase; there is no phase boundary between them for α_4 larger than the value at the CP. The phase labeled by “SCC” is the solitonic chiral condensate characterized by three parameters b , k and ν as: $\sigma(z) = k\nu^2 \text{sn}(b, \nu) \text{sn}(kz - b/2, \nu) \text{sn}(kz + b/2, \nu) + k \frac{\text{cn}(b, \nu) \text{dn}(b, \nu)}{\text{sn}(b, \nu)}$. It provides a one-parameter family of solutions to the equation $\frac{\delta\Omega}{\delta\sigma(z)} = 0$ [3]. “SPC” is the solitonic charged pion crystal condensate, a charged pion analog to the SCC state. It is defined by $\sigma \neq 0$ and $\pi_1(z) = k\nu \text{sn}(kz, \nu)$. In this phase, the charged pion component is oscillating in the homogeneous sea of scalar condensate. The SPC and Wigner phases are separated by a continuous second-order phase transition. At the phase boundary, a

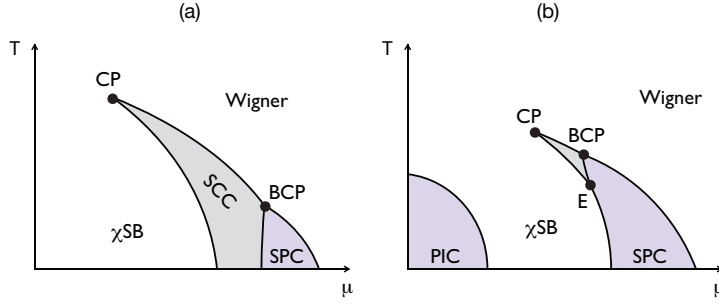


Fig. 2 Schematic picture of QCD phase diagram for $\mu_I \neq 0$: (a) for small μ_I^2 which shares the same topology with Fig. 1(a), and (b) for large $\mu_I^2 > m_\pi^2$, where the situation corresponds to Fig. 1(b).

charged pion mode at a finite wavevector becomes unstable; the amplitude of the mode grows exponentially with time. Nonvanishing μ_I^2 is responsible for this because it gives a negative contribution to the gradient term $(\nabla \pi_\perp)^2$ for $\beta_{4c} = -\frac{4}{3}\mu_I^2\alpha_6 < 0$. By contrast, the phase transition between the SPC and SCC phases is first-order. As a consequence, there is a bicritical point denoted by “BCP” where two second-order phase transitions and a first-order phase transition meet up. A notable change in the topology of phase diagram (b) from (a) is the appearance of continent of PC in the depths of homogeneous χ SB phase. This is triggered by the cross-term $\alpha_4\sigma^2\pi_\perp^2$ in the gGL potential Eq. (2) which drives the instability in the charged pion mode when $\alpha_4 < 0$ and σ^2 is large.

Let us finally draw a schematic QCD phase diagram in (μ, T) -plane, though it stays at the conceptual level. This is done in Fig. 2 where (a) and (b) are mapped into from those in Fig. 1. We utilized following two guidelines: (1) The topology in gGL phase diagram in (α_2, α_4) -plane should be kept. (2) In the vicinity of CP, the α_4 (α_2) axis points to higher T and lower (higher) μ direction [2]. In both cases (a) and (b), going up in density the system goes through the SPC phase before realizing an entire restoration of chiral symmetry. CP moves towards higher chemical potential and lower temperature with increasing μ_I . As soon as μ_I surpasses the mass of charged pion, the PC would be realized even in the absence of net quarks. For this reason the PC continent in (α_2, α_4) -plane is mapped on to the area including the origin.

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